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Refraction in the fixed direction at the surface of dielectric/silver superlattice

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Abstract

We report a novel abnormal refraction, called as refraction in the fixed direction, that occurs for certain dielectric/silver superlattices, in which the ray direction of the refracted wave is perpendicular to the surface of dielectric/silver superlattices, independent on the angle of incidence.

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Keywords: Superlattice; Refraction in the fixed direction

Since the experimental demonstration of the negative refraction [1,2] of microwaves in a two-dimensional array of repeated unit cells of copper stripes and split ring resonators by Shelby, Smith and Schultz [3], propagating properties of electromagnetic and optical waves in dielectric/metal periodic structures [4] received a great attention from many researchers [5–11].

In this Letter, we report a novel abnormal refraction of optical waves at surfaces of dielectric/silver periodic multilayer structures or superlattices, that we called refraction in the fixed direction, in which the ray direction of the refracted wave is perpendicular to the surface of the superlattice, independent of the incident angle.

The structure of the superlattice under consideration is schematically illustrated in Fig. 1. The superlattice placed in the region with $z > 0$ is formed by adjacent layers of a dielectric with the relative permittivity ε_1 and the thickness d_1, and silver with the relative permittivity ε_2 and the thickness d_2.

We consider transverse magnetically (TM) polarized plane waves that refract at the surface of the superlattice at $z = 0$. It is convenient to describe a TM polarized wave by using the magnetic field

$$\vec{H}(x, y, z, t) = u(z)e^{ik_x x}e^{-i\omega t}\vec{e}_y. \tag{1}$$

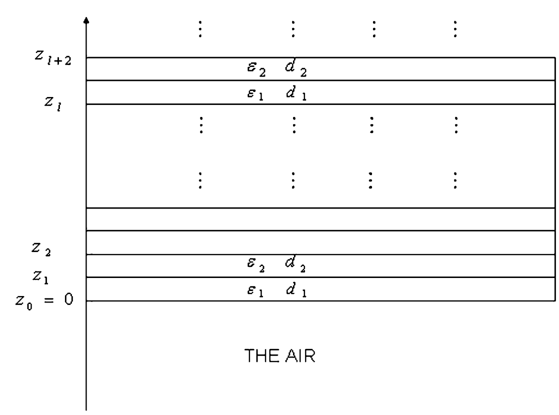

Fig. 1. The structure of a dielectric/silver superlattice.

The wave equation for the magnetic field $\vec{H}$ can be solved by using the transfer matrix method [12–14]. We have

$$u(z) = \begin{cases} e^{ik_{z0}z} + re^{-ik_{z0}z}, & \text{for } z < 0 \\ A_l \cos[k_{zl}(z - z_l)] + B_l \frac{\varepsilon_l}{k_{zl}} \sin[k_{zl}(z - z_l)], & \\ \quad \text{for } z_l \leqslant z < z_{l+1}, & \end{cases} \tag{2}$$

with $A_0 = 1 + r$, $B_0 = ik_0(1 - r)$, $k_{z0}^2 = k_0^2 - k_x^2$, $k_{z(2m+1)}^2 = \varepsilon_1 k_0^2 - k_x^2$, $k_{z(2m)}^2 = \varepsilon_2 k_0^2 - k_x^2$, and $k_0 = \omega/c$. The following

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recursion relations hold for coefficients A_l and B_l:

$$\begin{pmatrix} A_{2m+1} \\ B_{2m+1} \end{pmatrix} = T_1 \begin{pmatrix} A_{2m} \\ B_{2m} \end{pmatrix}, \quad \begin{pmatrix} A_{2m+2} \\ B_{2m+2} \end{pmatrix} = T_2 \begin{pmatrix} A_{2m+1} \\ B_{2m+1} \end{pmatrix}, \tag{3}$$

where

$$T_1 = \begin{pmatrix} \cos\theta_1 & \frac{\varepsilon_1}{k_{z1}}\sin\theta_1 \\ -\frac{k_{z1}}{\varepsilon_1}\sin\theta_1 & \cos\theta_1 \end{pmatrix}, \quad T_2 = \begin{pmatrix} \cos\theta_2 & \frac{\varepsilon_2}{k_{z2}}\sin\theta_2 \\ -\frac{k_{z2}}{\varepsilon_2}\sin\theta_2 & \cos\theta_2 \end{pmatrix}, \tag{4}$$

with $\theta_1 = k_{z1}d_1$, $\theta_2 = k_{z2}d_2$. The periodicity of the superlattice and the condition at infinity imply that $(A_0, B_0)^T$ must be an eigen vector of the transfer matrix $T = T_2T_1$, corresponding to an eigen value with a modulus less or equal to 1.

The eigenvalue μ of the transfer matrix T satisfies the following equation

$$\mu^2 - \left[2\cos\theta_1\cos\theta_2 - \left(\frac{\varepsilon_1 k_{z2}}{\varepsilon_2 k_{z1}} + \frac{\varepsilon_2 k_{z1}}{\varepsilon_1 k_{z2}}\right)\sin\theta_1\sin\theta_2\right]\mu + 1 = 0. \tag{5}$$

For a superlattice, the period $d = d_1 + d_2$ is much smaller than the wavelength. So for a large range of angle of incidence, we have $\theta_1 \ll 1$ and $\theta_2 \ll 1$. Thus Eq. (5) can be approximately written as

$$\mu^2 - 2\cos\theta\mu + 1 = 0, \tag{6}$$

where, to the second order of $k_0 d$,

$$\theta = \left[k_{z1}^2 d_1^2 + k_{z2}^2 d_2^2 + \left(\frac{\varepsilon_1}{\varepsilon_2}k_{z2}^2 + \frac{\varepsilon_2}{\varepsilon_1}k_{z1}^2\right)d_1 d_2\right]^{1/2}. \tag{7}$$

The solutions of Eq. (6) are $\mu_{1,2} = e^{\pm i\theta}$. To obtain propagating wave in the superlattice, we must have

$$T\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = e^{i\theta}\begin{pmatrix} A_0 \\ B_0 \end{pmatrix}, \tag{8}$$

that implies

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \tilde{t}\begin{pmatrix} 1 \\ \frac{i\theta}{\varepsilon_1 d_1 + \varepsilon_2 d_2} \end{pmatrix}, \tag{9}$$

where $\tilde{t}$ is the amplitude transmittance. Let $k_z = \theta/d$, we may write, for $z > 0$

$$\vec{H}(x, y, z, t) = e^{i(k_x x + k_z z)}e^{-i\omega t}\vec{e}_y\tilde{u}(z), \tag{10}$$

where $\tilde{u}(z) = u(z)e^{-ik_z z}$ is a periodic function. According to this expression, k_z can interpreted as the z component of an effective wave vector. In conformity with the expression (7), we have the following relation between k_x and k_z

$$k_x^2\left(\frac{\eta_1}{\varepsilon_1} + \frac{\eta_2}{\varepsilon_2}\right) + \frac{k_z^2}{\varepsilon_1\eta_1 + \varepsilon_2\eta_2} = k_0^2, \tag{11}$$

with $\eta_1 = d_1/d$ and $\eta_2 = d_2/d$.

The direction of the refracted ray in the superlattice is determined by the Poynting's vector averaged over two adjacent layers. By direct calculation, we found

$$\left.\begin{aligned} \langle S_z\rangle &= \tfrac{1}{2}|\tilde{t}|^2(\varepsilon_1\eta_1 + \varepsilon_2\eta_2)^{-1}\tfrac{k_z}{\omega} \\ \langle S_x\rangle &= \tfrac{1}{2}|\tilde{t}|^2\left(\tfrac{\eta_1}{\varepsilon_1} + \tfrac{\eta_2}{\varepsilon_2}\right)\tfrac{k_x}{\omega} \end{aligned}\right\}. \tag{12}$$

Then the angle of refraction is given by

$$r = \tan^{-1}\frac{\langle S_x\rangle}{\langle S_z\rangle} = \tan^{-1}\left[\left(\frac{\eta_1}{\varepsilon_1} + \frac{\eta_2}{\varepsilon_2}\right)(\varepsilon_1\eta_1 + \varepsilon_2\eta_2)\frac{k_x}{k_z}\right]. \tag{13}$$

For a dielectric/silver superlattice, we have $\varepsilon_1\varepsilon_2 < 0$. Thus by suitably choosing η_1 and η_2, we may have $\eta_1/\varepsilon_1 + \eta_2/\varepsilon_2 = 0$. In this case, indifferent to the angle of incidence, the angle of refraction always equals to zero. We call this phenomenon as refraction in the fixed direction.

As a demonstration of the refraction in the fixed direction, we carried out numerical calculations for refraction of optical beams with finite widths at the surface of a superlattice with $\varepsilon_1 = 4.8$, $d_1 = 24$ nm, $\varepsilon_2 = -4$ and $d_2 = 20$ nm, and presented numerical results in Fig. 2. The condition of $\eta_1/\varepsilon_1 + \eta_2/\varepsilon_2 = 0$ is satisfied for this superlattice. The following form for incident beam, in the free space, was used

$$\vec{H}^i(x, y, z) = \int dk_x \exp\left\{-\frac{\sigma^2}{2}\left[(k_{x0} - k_x)^2 + \left(k_{z0} - \sqrt{k_0^2 - k_x^2}\right)^2\right]\right\} \times \exp\left[i\left(k_x x + \sqrt{k_0^2 - k_x^2}z\right)\right]\vec{e}_y. \tag{14}$$

The parameter σ is set to 200 nm, corresponding approximately to a beam width of 1 µm. The wavelength of the incident optical beam is 400 nm. The incident beam can be treated as a superposition of plane waves with different wave vectors, and the magnetic field inside the superlattice is calculated as a superposition of refracted plane waves, by using the relations (2)–(4) and (9). Moduli of real parts of the magnetic field are presented in grey scale in Fig. 2. The darker color corresponds to the higher value. Two different angles of incidence, $i = 30°$ and $i = 60°$ were considered. We found that the ray direction of refracted wave is perpendicular to the surface of the superlattice in both cases. In other words, the refracted ray propagates in the fixed direction. The phase distribution can also be determined from the variation of the real part of the magnetic field. We also observe that, unlike the case of propagation in free space, the width of the Gaussian beam is unchanged along the direction of propagation.

The phenomenon of refraction in fixed direction can be explained in the following way: as the thickness of each layer of the superlattice is much smaller then a wavelength, the superlattice behaves as an effective homogeneous uniaxial medium at optical frequencies [15]. According to the symmetry of the superlattice, the optical axis is in the z direction. In conformity with the expression (11), we have for this effective medium,

$$n_o^2 = \varepsilon_1\eta_1 + \varepsilon_2\eta_2, \qquad n_e^2 = \left(\frac{\eta_1}{\varepsilon_1} + \frac{\eta_2}{\varepsilon_2}\right)^{-1}. \tag{15}$$

The condition for refraction in the fixed direction $\eta_1/\varepsilon_1 + \eta_2/\varepsilon_2 = 0$ is equivalent to the condition $n_e = \infty$, and in this case the ray velocity [16] is always in the direction of optical

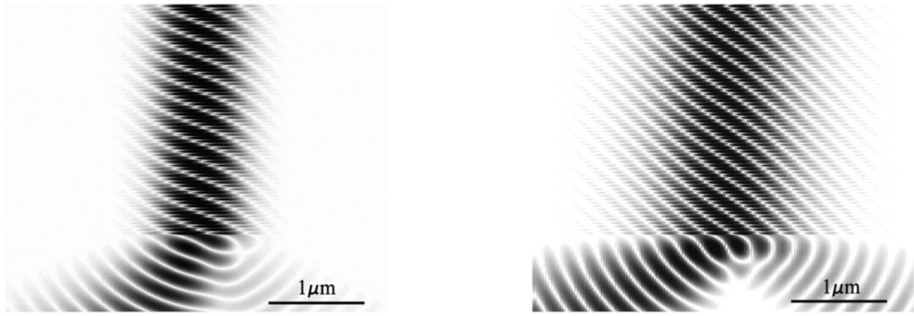

Fig. 2. Moduli of real parts of the magnetic field intensity of TM polarized waves refracted by a superlattice with $\varepsilon_1 = 4.8$, $\varepsilon_2 = -4$, $d_1 = 24$ nm, $d_2 = 20$ nm, $\sigma = 200$ nm, wavelength $\lambda = 400$ nm. Left: the angle of incidence $i = 30°$. Right: $i = 60°$.

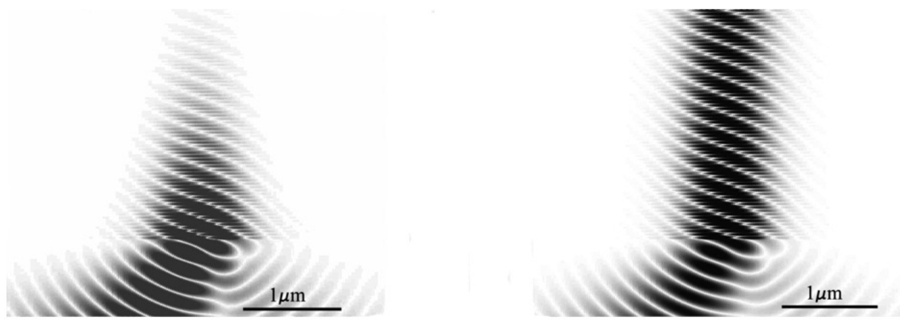

Fig. 3. Moduli of real parts of the magnetic field intensity of TM polarized waves refracted by a superlattice with $d_1 = 24$ nm, $d_2 = 20$ nm, $\sigma = 200$ nm, wavelength $\lambda = 400$ nm, and an angle of incidence $i = 30°$. Left: with losses $\varepsilon_1 = 4.8$, $\varepsilon_2 = -4 + 0.4i$. Right: with losses and an optical gain $\varepsilon_1 = 4.8 - 0.3i$, $\varepsilon_2 = -4 + 0.4i$.

axis, that is perpendicular to the surface of the superlattice in the present case.

So far we have treated the relative permittivities ε_1 and ε_2 as real numbers. But in reality, the relative permittivity of silver has a imaginary part in the order of 0.4 at wavelengths near 400 nm. To include imaginary parts of relative permittivities into consideration, we need just to replace $\eta_1\varepsilon_1 + \eta_2\varepsilon_2$ by $\mathrm{Re}(\eta_1\varepsilon_1 + \eta_2\varepsilon_2)$, and $\eta_1/\varepsilon_1 + \eta_2/\varepsilon_2$ by $\mathrm{Re}(\eta_1/\varepsilon_1 + \eta_2/\varepsilon_2)$ in expressions (12) and (13). The phenomenon of refraction in the fixed direction can still take place, but the condition becomes $\mathrm{Re}(\eta_1/\varepsilon_1 + \eta_2/\varepsilon_2) = 0$. When the imaginary part of permittivity of (or the optical losses in) silver is considered, the intensity of optical wave in the superlattice decreases along the z direction. This attenuation can be compensated if the dielectric is an optical gain medium [17]. Numerical results for the refraction in the fixed direction in the presence of optical losses and optical gain compensation are presented in Fig. 3. The impact of the losses in silver and the effect of the optical gain compensation can be easily observed from these numerical data.

In conclusion, we showed that the refraction in the fixed direction may take place at the surface of certain dielectric/silver superlattices. In the case of the refraction in the fixed direction, the ray direction of the refracted wave becomes independent on the angle of incidence, and is always perpendicular to the surface of dielectric/silver superlattice. By carrying out numerical calculations, we also observed that the width of optical beam remain unchanged within this kind of superlattice. The impact of the losses in silver and the effect of the optical gain compensation are also discussed.

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