$$Q_{rad} = \frac{\mathrm{Re}[x]}{2\,\mathrm{Im}[x]} = \frac{n_1^{\,2}(n_1^{\,2} - n_2^{\,2})^{1/2}\,\mathrm{Re}[x]}{2n_2^{\,2}\exp[-2T]} \tag{A3}$$

where x is the size parameter $x = 2\pi R/\lambda$. T is a variable that depends on x weakly; the radiative quality factor Q_{rad} therefore scales linearly with the size parameter:

$$Q_{rad} \propto x \tag{A4}$$

To match the lasing modes observed experimentally with theoretical WGM values, we derived the asymptotic solutions for mode order p and mode number m following [29]:

$$\lambda^{-1}(R, n_1, n_r, p, m) = \frac{1}{2\pi R n_1}\Big[m + \frac{1}{2} + 2^{-1/3}\alpha(p)(m+\frac{1}{2})^{1/3} - \frac{L}{(n_r^{\,2}-1)^{1/2}} + \frac{3}{10}2^{-2/3}\alpha^2(p)(m+\frac{1}{2})^{-1/3} - 2^{-1/3}L(n_r^{\,2} - \frac{2}{3}L^2)\frac{\alpha(p)(m+\frac{1}{2})^{-2/3}}{(n_r^{\,2}-1)^{3/2}}\Big] \tag{A5}$$

where $n_r = n_1/n_2$, $L = n_r^{-1}$ for TM modes, and $L = n_r$ for TE modes. $\alpha(p)$ are the roots of the Airy function.

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